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On the calculus of risk in construction projects: Contradictory theories and a rationalized approach

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ABSTRACT

Evaluating risks through numbers-although an inevitable stage of risk management-can be seriously problematic, especially when marginalized groups of risks turn out to be significant, for example, to the lives of people. While it questions the effectiveness of the traditional approach to risk scoring, the literature provides no alternative satisfying all the criteria stressed by the critics. In fact, different dimensions of uncertainty, along which a risk can be viewed, entail different quantifications. Most previous work, however, concentrates on supposedly all-purpose solutions that are often justified or promoted over others by reasons not necessarily applicable; little information is available on how to best select the needed scoring approach. This research investigates the issues involved in constructing a risk factor formula that is more consistent with the nature of the project and its goals. Major concerns addressed in the literature are organized, serving as a basis to evaluate and improve seven groups of alternative formulas in light of mathematical arguments without which fallacious conclusions-such as the myth that importance is implied by exponents greater than one-would be inferred. These groups are complemented by a multifaceted approach introduced for the first time in this paper, providing the observer with customized information about risks. A robust scoring system founded on these results will ensure that allocated risk factors are neither too high nor too low. Although expressed in the terminology of construction safety, the findings of this research can be extended to other industries that feature some element of uncertainty.

1. Introduction

It is the attitude of an organization toward uncertainty that determines how it will overcome potential failures. Improper treatment of uncertainty results in defective risk assessments and, thus, faulty decisions (Zio and Aven, 2013). Exemplary disasters that resulted from perceived but underrated risks can be found in the history of engineering, the analysis of which reveals a fundamental misunderstanding of different aspects of uncertainty.

Although it can be quite unrealistic in the absence of accurate information (Ale et al., 2015; Zio and Aven, 2013), a proper quantification of uncertainty is essential for the comprehension, description, and communication of the risks associated with a system under consideration, and how they change over time and after intervention (Apostolakis, 2004; Duijm, 2015; Mackenzie, 2014).

To many of those involved in risk management and research, the quantification process is driven by the relative seriousness of risks (Fine, 1971), expressed as an index or factor which has attracted attentions in recent years even more than what has been paid to the analyzed risks (Mackenzie, 2014), because it is the only way to identify priority risks (Groso et al., 2012).

Summarizing available information into a single number is indeed a difficult and sensitive issue (Mackenzie, 2014), which requires an analyst to carefully select and utilize constituent elements and algebraic operations (Azadeh-Fard et al., 2015; Ni et al., 2010). While nearly all the improvements or alternatives available to the traditional risk scoring formula have taken a ‘one size fits all’ approach, items such as the cause of uncertainty, properties of available information, and details required by the observer can determine which mathematical expression is best suited for a risk assessment tool (Groso et al., 2012; Zimmermann, 2000).

A widespread belief that risk is nothing more than the “expected loss” summarized by averaging the “probability” of events times their corresponding “impacts” can falsely relieve the effort required to

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manage unbearable risks which are, although low in the product of their probability and impact, high in other remarkable contents of uncertainty. Even though this traditional method of risk scoring looks simple and produces consistent results, a number of flaws identified in the literature suggest that its use should be revised (Bowles, 2003). The literature is, however, replete with both incautious uses and sharp criticisms of the product formula with no established criteria for determining when it is appropriate to be used. Shortcomings of the product formula are endlessly enumerated in the papers presenting new approaches which themselves suffer from the same problems.

The appropriateness of a risk scoring technique is usually examined from two perspectives: whether special contents of interest are addressed or not, and whether an acceptable distinction between important and unimportant risks is provided. The traditional two-dimensional risk calculation method is perhaps not capable of including parameters such as manageability, criticality, worsening factors, social amplification, voluntariness, dread, and familiarity (Ale et al., 2015; Derby and Keeney, 1981; Duijm, 2015; Groso et al., 2012; Kasperson et al., 1988; Zeng et al., 2007), and the traditional approach to combining the selected factors into a single factor, which uses the multiplication operator, is likely to produce unreliable results (Bowles, 2003; Duijm, 2015; Kaplan et al., 1981; Seyed-Hosseini et al., 2006; Williams, 1996).

This paper attempts to explain why and on what grounds the traditional method is continuously undermined, and how it can be improved or replaced. A large number of articles, including both original research and reviews, along with their references and the articles referring to them have been studied to illuminate the fundamental concerns of the critics. Not all criticisms are found to be based on accurate and relevant assumptions, nor do they apply to all types of risk scoring formulas. Major concerns addressed in the literature are first discussed, and summarized in six points detailed in Section 2. Amendments are also made, as appropriate. New approaches to risk scoring are then sought in the literature and assessed with respect to these points. These are introduced in Section 3 referring only to the oldest or most famous works that have implemented the suggested approach. A few approaches that have been found defeating their purpose are further scrutinized in the light of mathematical theories and examples. In Section 4, a completely new approach to risk scoring is presented. Comparing the attributes of different approaches introduced, a discussion is provided in Section 5 to clarify where to use each approach, and is further supplemented by a few case examples in Section 6. The results are then concluded in Section 7.

2. Concerns addressed in the literature

Rather than giving an overview of different approaches to risk scoring, this section first presents the major concerns addressed in the literature with the traditional risk scoring method, such that a subsequent overview of alternative approaches can adopt these concerns as a basis for assessment.

2.1. Dissimilarity

A most important and widely recognized drawback of risk scoring systems is the possibility of assigning similar Risk Factors (RFs) to naturally different risks. In this regard, critiques frequently found in the literature are categorized as the following (Ale et al., 2015; Bowles, 2003; Derby and Keeney, 1981; Duijm, 2015; Kaplan et al., 1981; Williams, 1996):

1. It is difficult to decide how to treat a risk based on a single RF, given that it provides no information about possible contributing factors. For example, if it results from the product of four and five, an RF of 20 only needs to be regularly monitored, but when the factors are ten and two, the higher factor should be reduced to a safer level while the lower one probably requires no action. When there are three factors multiplied together, the number 36, for example, is obtained from five different combinations: {(4,3,3),(6,3,2),(6,6,1),(9,2,2),(9,4,1)}, none of which is to be treated like the others.
2. Combining multiple factors into one takes no notice of their inequalities, i.e., the RF does not indicate whether either the Probability (P) or the Impact (I) is greater. Therefore, calculated RFs for a risk with high P and small I might be quite similar to one with low P and large I. To give an example, a potential threat to the lives of 100 people with a chance of one in a thousand and an unsafe act with a ten percent chance of claiming one life, although quite different in nature and features, will be assigned identical RFs. Yet the former calls for extensive design considerations and contingencies, while the latter can be eliminated by better education and more stringent regulations.
3. One can view the only purpose of risk scoring as to compare the risks, not to provide solutions. However, it is ambiguous even for someone who wants to compare the risks to see how some chains of inequalities ($RF_1 < RF_2 < RF_3$) show conflicting results. The product of 10×2, which implies an extreme case of both parameters, for example, is bounded between two moderate combinations 4×4 and 5×5.
4. A mysterious class of events often referred to as catastrophes result in such a heavy damage that they are usually expected to be flagged as important, almost regardless of how infrequent or irregular they are. Nevertheless, even maximal values of I will be overlooked when multiplied by a low P, therefore, many guidelines make specific reference to what they call 'risk aversion' or more specifically 'major risk aversion', suggesting that all the risks that are large enough in I should be manually assigned a high RF before they are multiplied by P. However, having ignored the fundamental element of uncertainty, P, this modification can also highlight some nearly impossible phenomena that happen rarely, if ever, only because they might be disastrous. Too much concentration on these unlikely but catastrophic risks at the expense of devising costly contingency plans will probably exhaust the resources required for other operations.

These problems, although often attributed to the product formula ($RF = factor1 \times factor2$), arise whenever a formula is symmetric on the variables, i.e. when it uses only 'commutative' operators, such as the product and average formula (Sections 3.1 and 3.2), or when non-commutative operators are used but interchanging the parameters does not change the results, as it is in the union-like formula (Section 3.3). While improvements such as those introduced in Sections 3.7 and 4 make the formula non-symmetric, taking the logarithm of a product (Section 3.6) is not beneficial in this regard.

2.2. Understandability

Systems are considered internally complicated if they are difficult to construct, and externally so, if they are difficult to understand (Ramasesh and Browning, 2014). While it is convenient to use an easy to calculate risk index such as those introduced in Sections 3.1–3.5, which may only require a desktop calculator, computer-assisted calculations (Sections 3.7 and 4) can be well worth employing to provide managers with less complicated and more readily understandable information.

Less information is not necessarily less complicated. Although results obtained from a single-output formula are better comparable (Ale et al., 2015), valuable information may be ignored when combining all the available data into a single number (Duijm, 2015; Kaplan et al., 1981; Williams, 1996). A single number gives no idea as to how the results can be improved, what a certain reduction in RF means, and whether a risk with an RF of 200, for example, is twice as risky as one with an RF of 100 (Bowles, 2003; Gilchrist, 1996; Mackenzie, 2014).

Multiple-output formulas discussed in Section 4 are therefore evolving to tailor the available information to satisfy the observer's needs. However, it cannot be denied that, despite the shortcomings noted in Section 3.2, most managers are already familiar with the traditional product formula, and as Mackenzie (2014) states, they must be trained to trust and react well to a newly proposed formula. Ale et al. (2015) believe that misunderstanding not only results in misleading interpretations, but also can be disastrous.

2.3. Reasonable sensitivity

When adjusting risk priorities, it is surprising to see how calculated RFs can be changed by a minor intervention in the contributing factors. For example, a risk assigned with an RF of 200 out of 1000, can be reduced to 100 by a one-point reduction in the third factor, if they are 10, 10, and 2. This level of sensitivity can thus make the RF swing around a predefined Acceptable Risk Level (ARL) just because an expert is not sure whether to set the third factor to one or two. However, if the RF of 200 is resulting from the three factors 8, 5, and 5, reducing the third factor to 4 will only decrease the RF to 160. This means that the sensitivity of the product formula to each factor is dependent on the size of other involving factors (Bowles, 2003).

Taking the logarithm of the product formula is often known to be able to solve the problem (Braband, 2004), although it does so only partially. As shown in Section 3.6, a logarithmic transformation is more sensitive to incremental changes when inputs are small than when they are large. Increasing the input from 1 to 2 (a one-point increase) has an effect similar to increasing it from 5 to 10 (a five-point increase), since it is the order of magnitude that counts. In return, the logarithmic transformation seemingly separates the effect of each factor from any others and remarkably stabilizes the sensitivity of the formula, as a single factor can affect the output only to a certain extent quasi-independent of the value of other involving factors.

2.4. Surjectivity

A number of authors have highlighted the problems of multiple 'holes' in the outputs of the traditional product formula. For example, if either of the two factors contributing to RF is 9 or they are both 10, the product formula discussed in Section 3.2 gives RFs of 90 or 100, and no other number between these two is achievable, i.e. there is a 10 point length hole between 90 and 100. This problem is more evident when there are three or more factors varying from 1 to 10, and the maximum possible three numbers are 1000, 900, and 810 separated by two large holes with the length of 100 and 90. Bowles (2003) views the holes as the most serious drawback of the product formula, and mentions a few problems that arise with holes, e.g., it is rather ambiguous to interpret the hole that appears between 64 and 70, while there is no hole between 63 and 64.

Despite the common concern with the number or size of the holes, these holes can be simply reduced when the inputs are obtained from fuzzy decision systems or averages of multiple expert opinions, which can take values not limited to integer numbers. Instead, it is the distribution of results that determines their appropriateness. The size of the hole between 900 and 1000 is not itself a problem but when compared to that of holes around 100, it reveals that the majority of results are accumulated at the low-risk end of the distribution (Müller et al., 2006). While only seven out of 1000 numbers resulting from the product of three 0 to 10 integer numbers are greater than 800, as an example, 710 numbers (71% of total) are less than or equal to 200. Thus, the product formula provides too much separation at the high-risk end, and too much density at the low-risk end of the spectrum.

Different formulas are proposed in the literature to overcome the problems of holes (see Ouédraogo et al. (2011a) for example) but none eliminate the presence of holes entirely. In other words, these formulas are only different in their distribution of results and its characteristics, such as its skewness, not the presence or absence of holes. The Probability Density Functions (PDFs) shown in Fig. 1 indicate how the distribution of results varies in a common range depending on the employed formula. It is noted in the next section that not all the formulas introduced in Section 3 produce outputs confined in the range [0,1], but according to Table 2 (Section 4) they can be modified to have similar ranges. To give a few examples, the solid (black) curve associated with the traditional product formula shows a large portion of results gathered in the low-RF zone, which can be slightly moderated by reducing the exponent of one of the factors to 0.5 shown by the dash dot (brown) curve. In contrast, the PDFs calculated from the logarithm and union-like formula are skewed negatively, suggesting that a larger percentage of risks are flagged high.

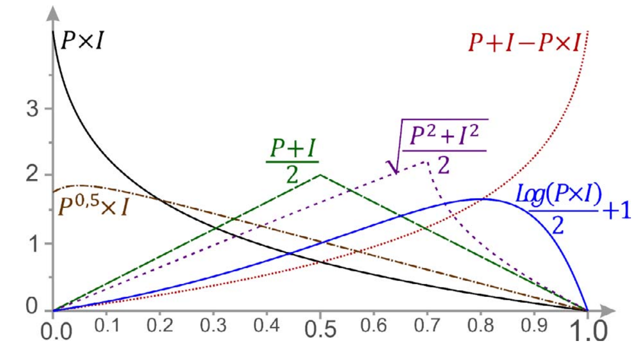

Fig. 1. The distribution of results obtained from a number of formulas suggested for calculating RF.

2.5. Confinement to a specific range

Numbers confined to a specific range are often better understood than those able to take unbounded values. Upper bounds are usually perceived as a measure with which calculated RFs are to be compared. A risk assigned with an RF of 80 is commonly considered tolerable if it is out of 1000 and intolerable if out of 100, but it makes little sense if the upper limit is not defined. Except for the logarithmic transformation, all the approaches introduced in Section 3 produce bounded outputs if they are fed with bounded inputs.

It is also of utility to clearly define the range of the function in use. The multiplication of two numbers between 0 and 10 is perhaps commonly acknowledged to range between 0 and 100, but a failure to notice that raising one of the parameters to the power of 1.5 or 2 extends the range to [0,316] or [0,1000] reinforces the myth that the parameter raised to a greater power is receiving more attention, an issue which is further explored in Section 3.7.

Risk visualization purposes usually require the RF to be expressed on a 0–1 (or 0–100%) scale, which can be usually obtained by dividing the RF to its maximum possible value, or according to Mackenzie (2014), to other values such as the maximum acceptable or desirable level of risk.

2.6. Monotonicity

Almost every author who defines a new formula to calculate RF indicates that it is monotone, i.e., increasing a parameter given that all the other parameters are constant increases the RF (see Braband (2006), Ouédraogo et al. (2011b), and Duijm (2015)). Except for the approaches that use 'division', such as the efficiency scoring methods discussed in Section 3.5, all the formulas introduced in Section 3 are monotonically increasing functions.

3. Possible approaches to risk scoring

Having presented and discussed major concerns in the literature with the traditional risk scoring method, summarized in six points, this section presents an overview of alternative approaches to risk scoring available in the literature, grouped by approach and each group assessed with respect to the previous six points.

3.1. A combination of P and I

The two basic parameters traditionally used to describe a risk -regardless of how they are labeled or measured- typically convey information about how imminent an event is, and whether it exerts a dominant effect on the project in the case that it happens. There is almost no dispute among authors that a risk with both higher P and larger I has to be ranked higher than one with both lower P and smaller I. Thus, it can be said that $RF \propto P$ and $RF \propto I$, which means that RF is related positively to both P and I. Assuming that P and I are measured in a ratio scale, possessing equal intervals and a definite zero point (see Bowles (2003)), arithmetic operations can be performed to reach a value for RF.

Perhaps the simplest combination to form an RF is the addition formula (Kaplan et al., 1981), that is $RF = P + I$, which can be rewritten as the average formula, $RF = \frac{P+I}{2}$, to confine RF to the range [0,1] when P and I are normalized to their maximum value. However, this formula has not generally been accepted since 'addends' need to share similar characteristics (Ni et al., 2010), and the addition or average of two numbers implies the sense that they represent two instances of the same concept. The sum of direct and indirect costs of occupational injuries, for example, meaningfully expresses the total cost, but it is completely vague to add this number to an environmental attribute like the concentration of carcinogens in the air, even if they are all normalized into the same range. Therefore, it is not necessarily acceptable to combine these two parameters in this way. Moreover, a risk with a maximum degree of P but zero I, or vice versa, will be allocated an average RF of 0.5 in this approach, which is not usually expected to be so.

3.2. Focusing on risks that are both high in P and large in I

The product formula $RF = P \times I$, which goes back to the 1660s (Ale et al., 2015) and is the most frequently chosen combination of P and I (Samson et al., 2009), presupposes that a risk is high if and only if it is high or large enough in both parameters. Scrutinizing this approach, one can see the possession of high P or large I as two virtual events, event $\overline{P}$ and event $\overline{I}$, the probability of which are equal to P and I, and the intersection of which yields the event of having high risk (see Fig.2a). The fault tree demonstration of this composition indicates that the top event 'risk' will not occur if any of the sub events are missing (see Fig. 2b).

This presupposition, however, has been widely criticized for ignoring major but rare hazards, which is extensively discussed in Section 2.1 (Dissimilarity). Furthermore, with reference to Probability Theory, the joint probability of multiple events is equal to the product of their individual probabilities, if and only if they are mutually independent, whereas there is no evidence that $\overline{P}$ and $\overline{I}$ are always so. Impacts are often found to be negatively or even positively correlated to their probabilities, and hence the multiplication of P and I cannot be supported by Probability Theory.

Not only do these fundamental questions disprove the conjecture that risk is constructed from the product of P and I, but there are also arguments against the properties of an index defined in this way. Neither does it provide a uniform resolution in low to high risk areas (Surjectivity, Section 2.4), nor does it reflect reasonably to small changes in inputs (Reasonable sensitivity, Section 2.3). Authors like Gilchrist (1996), Cooper et al. (2005), Ni et al. (2010), and Duijm (2015) believe there is no inherent logic to multiply the basic parameters P and I, while they suggest that other mathematical operations might be of better utility. It should be noted however that the product formula can still be a convenient tool for determining statistical quantities such as the expected loss.

3.3. A semi-selective approach

In contrast with the highly selective approach presented in the previous section, i.e., the product formula, Cooper et al. (2005) recommend that a risk should be flagged as important if it has either high P or large I. In this approach, all risks are to be considered high, unless they are low or small enough in both parameters. Continuing with virtual events $\overline{P}$ and $\overline{I}$ defined in the previous section, one can consider the state of having low P as event! $\widehat{P}$ with the probability of $(1{-}P)$, and similarly, the state of having small impact as event $\widehat{I}$ with the probability of $(1{-}I)$. If events $\widehat{P}$ and $\widehat{I}$ are assumed independent, events $!\widehat{P}$ and $!\widehat{I}$ will also be independent, thus the joint probability of having both low P and small I can be written as $(1{-}P) \times (1{-}I)$, and if subtracted from 1, the probability of not having low P nor small I will be $1{-}(1{-}P) \times (1{-}I)$ which yields to $RF = P + I{-}P \times I$ when simplified. Fig. 3b shows how the top event 'risk' occurs if either of the basic events occurs. This 'Union-Like' formula is also capable to address three or more parameters, say X, Y, and Z, by simplifying the phrase $RF = 1{-}(1{-}X)(1{-}Y)(1{-}Z)$ to $RF = X + Y + Z{-}XY{-}YZ{-}XZ + XYZ$.

Cooper et al. (2005) believe "it is better to have [listed] too many risks than too few", but it may not be the case in a very complicated project, as it makes managers prone to the 'cry wolf' effect (see Oboni and Oboni (2012)). To give an example, taking into account the union of events $\widehat{P}$ and $\widehat{I}$ (Fig. 3a) will allocate both a meteorite able to destroy the whole project $(I \simeq 1)$, and a small puddle frequently found in a construction site $(P \simeq 1)$ the highest RF, i.e. $RF \simeq 1$, although quite negligible is P in the former and I in the latter. A high level of judgment by both the analyzer and observer is hence required to prevent irrational risks from piling up in the priority list, nevertheless, it should be acknowledged that this approach cautiously avoids any possibility of ignoring important risks.

3.4. Involving a third factor

Among the motivations for this research is the fact that a 'time

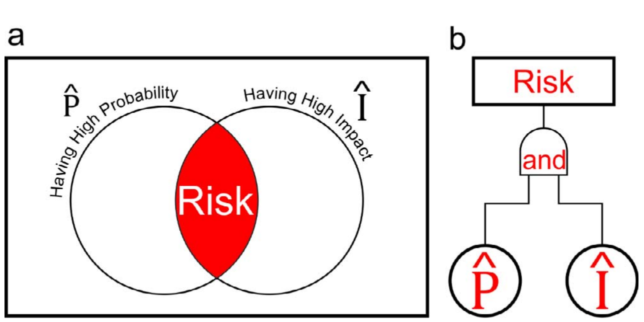

Fig. 2. (a) The intersection of two sets, (b) Basic events connected via 'and gate'.

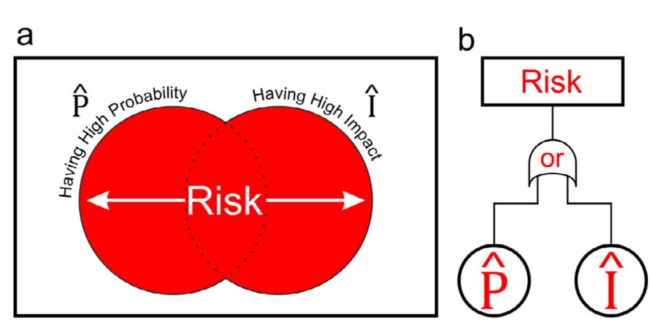

Fig. 3. (a) The union of two sets, (b) Basic events connected via 'or gate'.

factor' (T) incorporated in a risk scoring system seemed very strange to the first author when he was invited to assess the risk management process of a major hospital construction. The introduced RF defined as $P \times I \times T$ was to decrease as the time available to manage the risk increased. For example, the output of $P \times I$ was suggested to be multiplied by $T = 0.4$ if the predicted risk event was not expected to occur in the next 12 months, and by $T = 1.0$ if likely to occur in 3 months.

It is certainly true, in some circumstances, that a risk expected sooner appeals more primarily to managers, but, final phases of projects are sometimes prone to major risks that must be treated at the beginning, and it is too risky to postpone the actions required to manage them simply because they are not urgent at the time of assessment. In other words, a project team cannot be presumed able to respond to every risk in a timely and effective manner upon being alerted. Nevertheless, involving a third factor may be meaningful. In practice, other aspects such as detectability presented in the following section are found to be of more importance than time.

3.4.1. The RPN used in FMEA

Many reliability studies employ Failure Modes and Effects Analysis (FMEA) to identify, assess, and provide solutions to possible problems in a system, as well as details about their causes and effects, and a Risk Prioritization Number (RPN) commonly defined as $RPN = P \times D \times I$, in which D denotes the probability that a failure mode remains undetected during normal operations or tests, in a way that the higher the probability of detection, the lower D and thus the lower RPN.

The questionable selectivity of the product formula (Section 3.2) is here increased as only those failure modes that are high or large enough in all three parameters will be flagged as important, and remediable risks are filtered out. In addition to the previously mentioned criticisms of multiplying the involving parameters (Section 2), a few disputes can be found about the efficacy of a 'detectability factor' considered exclusively. Zhao et al. (2013), for example, deem it embeddable in the cost (impact) factor, and replace D with other parameters in their work. Bowles (2003) states that expert opinions concerning the 'detectability' can be highly subjective, the name is misleading, and it is not reasonable to lower the priority of detectable risks, especially when it makes people think they can fix detectable design flaws later.

Furthermore, while FMEA's RPN works fine in process and manufacturing industries, where the whole production line can be stopped to capture a faulty product or deviation before it is too late, it is not always enough in the construction industry to know that something wrong is going on. Enormous, unpreventable risks-whether detectable or undetectable-will undoubtedly have important consequences. The risk of a dam overtopping, for example, was before the 1960s believed to be effectively controllable by lowering the water level to some ten feet below the dam crest, but when the accelerated movements of the Vajont Dam's right bank warned of an imminent landslide, no one was able to prevent it from pushing billions of gallons of water over the dam (Kilburn and Petley, 2003; Mantovani and Vita-Finzi, 2003). This third factor 'D', together with the one discussed in the previous section (T), can be in the general form coupled with another factor 'Unpreventability' presented in the next section to form a more influential factor.

3.4.2. Presuming all controls are in place

New projects are always associated with the fear of failure due to some potential risk that is probably outside the host organization's control. Such issues can be better analyzed if properly brought to light by a specific risk scoring system. Azadeh-Fard et al. (2015) suggest multiplying the calculated score by a new parameter Unpreventability (U) to obtain a Residual RF (RRF), which is increased up to 50% if the perceived risk is definitely unpreventable, or decreased up to 50% if it can be simply prevented by available measures. Mathematically speaking, $RRF = P \times I \times U$, in which U is a real number between 0.5 and 1.5. One can regenerate the formula as $RRF = \frac{3}{2}P \times I \times U$ to normalize the results to the range [0,1], or define U on a zero to one scale, i.e. 0.0 for risks that can be prevented 'totally', 0.5 for 'partially' and 1.0 for 'by no means'. Involving such a factor however cannot be routinely recommended, especially when the risk prioritization process is to direct the focus of risk treatment activities, as it can result in a number of preventable, but mistakenly not prevented risks damaging the organization's reputation.

3.5. Risk prioritization and then what?

The prioritization process is usually associated with an Acceptable Risk Level (ARL) measured in the same scale as RF. All identified risks assigned with an RF greater than ARL are considered so intolerable that they must be treated. For example, an ARL of ten or fifteen percent of the maximum value of RF can be suitable (Müller et al., 2006), that is, when risks are scored from 0 to 1 those with RFs greater than 0.15 will be planned to be dealt with, while those with lower RFs are assumed safe to be tolerated.

ARL is not an inherently satisfactory or ideal level, but the lowest possible level less than which seems to be unachievable (Derby and Keeney, 1981), and there is no scientific basis to generally prescribe a specific ARL (Ale et al., 2015). Furthermore, it is not always clear whether RF is to be reduced by altering P or I (Dissimilarity/Understandability, see Sections 2.1 and 2.2). Williams (1996) finds risk rankings unreliable and recommends considering both P and I of all risks at all times. Bowles (2003) describes the whole process of calculating RFs, comparing them with a subjective ARL, and trying to cut down higher RFs as an 'unproductive numbers game', from which no clue can be obtained as to how the risks should be managed.

Some authors argue that, therefore, risks should be ranked in a way that rationalizes the subsequent treatment activities, which can be measured based on their efficiency in terms of the loss they reduce divided by the resources they consume (Cox, 2009; Mackenzie, 2014; Ni et al., 2010). The justification equation suggested by Fine (1971), for example, takes into account the whole formation path of information about an event to resource allocation decisions. He first defines RF as the product of P, I, and a third parameter Exposure (Ex), then expands the approach by introducing a Justification factor (J) to consider a responsive action more justifiable if it costs less and corrects more. It is defined as $= \frac{P \times I \times Ex}{CF \times DC}$, in which CF stands for the Cost Factor and DC for the Degree of Correction. Risk treatment options must then have small denominators to be selected.

However, this approach to justification is indeed a poor pretext to abandon contingency plans especially when it comes to safety, or the viability of an organization is at stake. To address this problem, Fine (1971) also suggests that an exception should be made for what he calls "highly hazardous situations".

Optimization techniques are also advocated (by Cox (2009) for example) to obtain more effective risk action plans. Risks threatening a complex system are often likely to have arisen from correlated or shared origins, thus selecting an optimized portfolio of actions will result in better outcomes. Seyed-Hosseini et al. (2006), for example, propose an algorithm to improve FMEA by considering all direct and indirect relationships between possible failure modes, and rank the more influencing failures higher than those influenced by the superior ones.

Although the selection of an optimized treatment portfolio is a must, it should be noted that risk treatment planning is not always the only purpose of risk prioritization. Moreover, a planning approach focusing on those treatment alternatives that should necessarily tackle multiple risks will fail to look after isolated but possibly enormous risks.

3.6. Focusing on orders of magnitude

Apart from the elements involved in the expressions cited above, possible approaches to their combination are basically those introduced in Sections 3.1–3.3, i.e. the average formula, the product formula, and the union-like formula. Yet, variants of the product formula are found in the literature overcoming (or at least claiming to have overcome) the limitations previously enumerated. The two commonly proposed improvements are raising elements to higher powers, which is described in the next section, and taking the logarithm of the product, described here.

Long-range parameters are often recommended to be expressed on a logarithmic scale, where small quantities cannot be ignored in favor of larger ones, and values at both ends of the spectrum are of interest. The energy an earthquake with a magnitude of 8 releases is one thousand times that of one with a magnitude of 5, though they are both considered significant. Instances of probabilities and impacts can be found in the literature categorized in logarithmically spaced groups, such as one, ten, one hundred, and one thousand fatalities resulting from a disastrous event. Instead of dealing with increments of quantities in such circumstances, a logarithmic scale can represent their differences in orders of magnitude. A lethality index can then be expressed, respectively, as zero, one, two, and three in this example.

While using a logarithmic scale is essential when data are basically logarithmic (distributed exponentially) (Duijm, 2015; Gilchrist, 1996), care should be taken when no such indication is given. Fig.4a shows how the outputs of a logarithmic function cluster around the maximum value when the inputs are equally spaced in the domain, which means that, in the context of risk scoring, almost all the risks will be allocated high RFs with a low separability, leading again to the 'cry wolf' effect. Fig.4b in contrast shows how well the outputs will be distributed when the inputs are spaced exponentially.

It is sometimes said that all the concerns addressed in Sections 2.1–2.6 are fully satisfied by simply taking the logarithm of RF. It is often stated that, for example, this transformation produces gapless results, a reasonable sensitivity to changes in the sub-factors, and an easily implementable and interpretable representation of RF because the logarithm of a product formula can be written as a simple summation (Braband, 2004; Groso et al., 2012; Müller et al., 2006; Ouédraogo et al., 2011a). However, as discussed earlier, gaps are not reduced but moved to the other end of the range, sensitivity to changes can be quite unfavorable when the inputs are not basically logarithmic (see Fig. 4a), and it is hard to accept that the combination of a logarithm and a summation is easier to calculate than a single multiplication. Furthermore, the results of a logarithmic transformation can be difficult to understand, as people may misinterpret the results as a basically linear set of data, and assume similar distances between the obtained numbers, while it is not so (Mackenzie, 2014).

Base-10 logarithms are frequently used because they are approximately related to the number of digits or decimals of the input. Nevertheless, Ouédraogo et al. (2011a) suggest that the bases can be reduced to $\sqrt{10}$, i.e. $RF = \log_{\sqrt{10}}[P \times I]$, presumably to increase the value of outputs, and alternatively, they can be set flexibly to reflect different weights of the involving parameters, i.e. $RF = \log_{b1}[P] + \log_{b2}[I]$, in a way that the greater the weight, the smaller the base. In addition to the cumbersomeness of these modifications, the desired improvements can be easily obtained, instead of considering strange bases, by setting appropriate coefficients in the equation, because $(\log_{b1}[P])$ is equal to $\left(\frac{1}{\log b1}\log[P]\right)$ or $(c1\cdot\log[P])$, which has the same effect as raising P to an exponent $c1$ discussed in Section 3.7.

Finally, it should be noted that logarithmic scales have one notable advantage, that is, they convert hyperbolic iso-risk contours to straight lines, and one serious disadvantage, that is, they can never accommodate zero probabilities or impacts (Levine, 2012). Moreover, while the logarithm has a domain of $(0,+\infty)$, its codomain is defined as $(-\infty,+\infty)$, which means that some hazards may be allocated negative RFs. When P and I are normalized to the range $(0,1]$, RF will fall in the range $(-\infty,0]$, and one can define it as $RF = 1 + \frac{1}{2}\log[P \times I]$ to map the results to the range $(-\infty,1]$

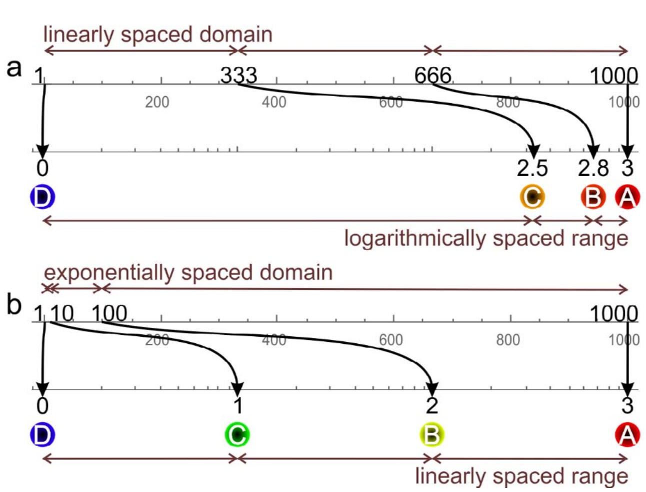

Fig. 4. The logarithm function mapping from (a) linearly and (b) exponentially spaced natural numbers to (a) logarithmically and (b) linearly spaced real numbers.

3.7. Rethinking the balance between P and I

When forming a formula, numbers raised to a higher power are normally supposed to be of more importance or, alternatively, the result is more sensitive to them. Since a most serious shortcoming of the product formula ($RF = P \times I$) is the fact that unexpected catastrophes are not properly distinguished (see Sections 2.1 and 3.1), the formula can be so tuned that large I risks are assigned a high RF, even though they might have a low P. The goal is, so to speak, to increase RF when P is low and I is large (see Fig. 5a).

Authors such as Okrent et al. (1981), Zio (2007), and Duijm (2015) mention that the parameter I can be raised to a power greater than one so that risks with larger I are assigned higher RFs. This modification, which is believed to ensure 'major risk aversion', however, does not show such an effect, which is better understood when the parameters are compared to their maximum possible value, or normalized and brought into the range [0,1].

Raising a number which is less than 1 to a greater power decreases the result rather than increasing it, and to be more precise, raising I to a greater power not only does not improve RF for highly destructive events, but even severely reduces it when I is medium or relatively small. Fig.5b shows that the shape of RF remains intact at the 'low P large I' region of interest, after increasing the exponent of I, confirming the uselessness of raising I to a power greater than 1. Instead, comparing Fig. 5c–a reveals that the desired result will be obtained by 'reducing the exponent of P', as it shapes a 'horizontal parabola' on the plane 'I = 1', providing greater values for RF when P is low and I is large.

Table 1 shows the general pattern of how the result will be affected by changing the exponents of P and I. The seventh row of Table 1 shows that incidents with large I and low P will be granted greater RFs when the exponent of P is slightly decreased. For demonstration purposes, reducing the exponent of P to 0.5, i.e. $RF = P^{0.5} \times I$ works properly to distinguish such incidents, but to examine an exact optimal value is beyond the scope of this research

Moreover, to have a formula with effectively tuned exponents, one can alternatively think of a weighted geometric mean, which always keeps the sum of all exponents equal to one. Therefore, the original form of a geometric mean of two numbers is $\sqrt{P \times I}$, and their weighted geometric mean will be $\sqrt[a+b]{P^a + I^b}$, in which a and b are the tuned exponents of P and I. For example, $\sqrt[4]{P \times I^3}$ displays an extreme case of risk aversion, which is of course the other form of $P^{0.25} \times I^{0.75}$

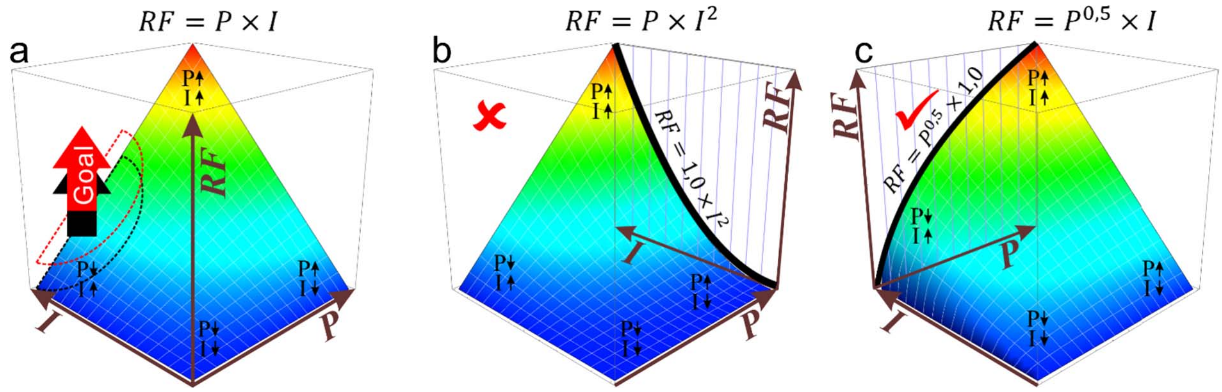

Fig. 5. The three dimensional representation of how RF can be affected by changing the exponents of P and I. (a) The original product formula, indicating the need to increase RF in the high P small I region. (b) The exponent of I is increased. RF is shaped as a parabola on the plane 'P = 1.0'. (c) The exponent of P is decreased. RF is shaped as a 'horizontal parabola' on the plane 'I = 1.0'.

Table 1
The effect of possible modifications to the risk scoring formula.

If	Is		For example	RF will be	Especially when	Which implies that
The exponent of I	Increased	Slightly	$RF = P \times I^2$	Decreased	I is small and P is high	Small impact events are inconsiderable, even if they are frequent
		Greatly	$RF = P \times I^5$	Extremely decreased	I is not too large	Risk will exist only if impacts are large enough
	Decreased	Slightly	$RF = P \times I^{0.5}$	Increased	I is small and P is high	Frequent events are of high risk, even if their impacts are small
		Greatly	$RF = P \times I^{0.2}$	Increased	P is high	RF depends only on probabilities, and impacts are ignored
The exponent of P	Increased	Slightly	$RF = P^2 \times I$	Decreased	I is large and P is low	Unlikely events are inconsiderable, even if their impacts are large
		Greatly	$RF = P^5 \times I$	Extremely decreased	P is not too high	Only those events that are almost certain can be considerable
	Decreased	Slightly	$RF = P^{0.5} \times I$	Increased	I is large and P is low	Large impact events are of high risk, even if they are not too frequent
		Greatly	$RF = P^{0.2} \times I$	Increased	I is large	RF depends only on impacts, and probabilities are ignored

4. A new approach to providing multifaceted information

Unknown phenomena are often analogized to an elephant being examined by blind people (or people in the dark), each of whom characterizes and describes the creature in a somewhat different way based on their limited individual perception (see Rayner (1987) and Ramasesh and Browning (2014), for example). Zimmermann (2000) states that while no unique, general definition of uncertainty is found in the literature, some have been claimed to be the only consistent one. According to Samson et al. (2009), almost all definitions of risk and uncertainty are introduced in response to a newly emerged problem, i.e. they are 'problem specific' rather than generic. Yet, regardless of the possibility of providing a generalized definition, it is good practice for construction managers to define problem specific RFs considering the phenomena being analyzed, involving variables, and outcomes of interest.

Managers are often seen, for example, to be so aware of high probability issues that they do consider appropriate measures addressing such issues as part of their normal planning. To give an instance, designers following a seismic code in a high seismicity region routinely behave responsibly toward the damage resulting from an earthquake, with no need to be aware of the background uncertainty. The impact resulting from frequently occurring events is therefore not the reason why organizations establish a risk management unit, but the mission of such a unit is to highlight, and provide solutions for events with more surprising outcomes.

A few authors like Müller et al. (2006) and Mackenzie (2014) have suggested that risks can be scored using a multiple-item index, and comment that failure to address all the items leaves valuable information unused. The idea presented in this section is to incorporate the ratio of the impact and probability of an event into the risk prioritization process, so as to measure the 'surprise content' of uncertainty as an auxiliary metric. By dividing I by P, a risk with large I and high P for which established guidelines are available will be of less surprise than one with large I but low P. To avoid the common problem of division by zero that arises when P is almost zero, the inverse trigonometric function 'arctangent' can be used to map the results from $[0,+\infty)$ to the range $\left[0,\frac{\pi}{2}\right]$. The surprise content is thus defined as $\varphi = Arctan\left(\frac{I}{P}\right)$.

When visualizing risk data sets on the Cartesian plane, P is sometimes placed on the horizontal and I on the vertical axis (Ni et al., 2010), and sometimes vice versa (Ale et al., 2015). Risks are then depicted as single points or vectors on the plane (see Fig. 6). As it can be seen from Fig. 6, φ is the angle subtended by the vector representing a risk with the dataset (p,i) and the positive P axis. Given that r is the 'radial distance' of the point to the origin, or alternatively the length of the vector, a new RF can be defined as the complex number $r\sphericalangle\varphi$, and to maintain the output in the range [0,1], one can divide the radial distance by $\sqrt{2}$ which yields $RF = \sqrt{\frac{P^2+I^2}{2}}\sphericalangle Arctan\left(\frac{I}{P}\right)$.

For example, considering $P_A = 0.8$, $I_A = 0.5$, $P_B = 0.3$, and $I_B = 0.9$ for two arbitrary events A and B, RF will be calculated as $RF_A = 0.67\sphericalangle 32^{\circ}$ and $RF_B = 0.67\sphericalangle 72^{\circ}$, respectively. Since complex numbers do not support linear ordering, it is not possible to compare the two events A and B based on their so defined RFs, but deliberate information can be inferred from this metric. In this example, whereas both events have an r of 0.67, the larger φ warns against the more surprising event B rather than the relatively ordinary event A.

Two areas of concern are distinguished in Fig. 6: First, events that fall into the red zone have both high P and large I, which are usually called 'extreme risks' that might already have enough resources reserved. On the other hand, events that fall into the dark purple area will probably cause a surprise when they occur, regardless of their r. A manager therefore has to take into account both r and φ: the distance r for estimating the magnitude of risk, and the angle φ for the level of surprise.

As described in Sections 3.4 and 3.5 there is a clear tendency to involve factors other than P and I. When three or more factors are

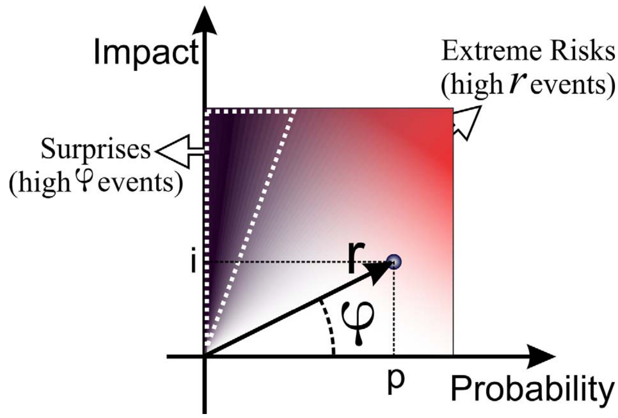

Fig. 6. The 2D visualization of risks in the Cartesian plane.

contributing to RF, potential information can be obtained from pairwise comparisons, a three-dimensional representation of which is illustrated in Fig. 7. The cube formed by the three coordinate factors X, Y, and Z is colored based on the length of the vector pointing to a risk in Fig. 7a (the radial distance formula), on the ratio of the factors Y and X in Fig. 7b, and Z and X in Fig. 7c, suggesting that risks falling into the red zone in each representation may require separate treatment programs.

5. Discussion

The quest for a Risk Factor is not a mathematical solution for a physical quantity; nor is it to provide a best estimation, or even an abstraction of a real world phenomenon. It should be regarded, instead, as an endeavor to systematically distinguish between risks that possess or lack a certain property, or to rank them according to their degree of possession of that property. No set of RFs can be therefore considered as a true value the deviation from which explains the weakness of an approach. They can only be evaluated by observing their attributes and the extent they fulfill the promised goals.

There are, of course, a number of well-rehearsed scoring approaches described in Section 3 with no indication of any consistent superiority of one approach over the others, albeit the literature seemingly insists on introducing a definite alternative, which indeed has not been agreed upon yet. Attributes usually pointed out to promote an approach over another are studied in Section 2. Dissimilarity, understandability and reasonable sensitivity (Sections 2.1–2.3) refer to the interests of an observer who uses the RF to make decisions in their uncertain environment, where the next three appear to be of less importance. Surjectivity, confinement, and monotonicity (Sections 2.4–2.6), in return, get more attention from the team who analyze the risks. Except for understandability, these attributes can be mathematically examined, which is summarized in Table 2.

It is shown that only the last two approaches, i.e., the Tuned exponents and the Multifaceted approach can produce dissimilar results for naturally dissimilar risks (Dissimilarity, see Section 2.1), and, only the average formula responds reasonably to alterations (Reasonable sensitivity, see Section 2.3). Since the logarithm of a product formula takes the form of a summation formula, operands in the logarithmic approach are limited in their influence on each other, thus the sensitivity of the result to a change in one variable is disconnected from the value of other variables. The multifaceted approach is reasonably sensitive in its magnitude part, but highly sensitive in the argument, because it involves the quotient of the mutual parameters.

Surjectivity and monotonicity are attributed to all approaches. With respect to confinement, the results of most of the formulas can be adjusted such that they cover only a desired range (see Section 2.5). As might be expected, taking the logarithm of, or dividing by a parameter that approaches zero will cause problems, which is reflected in the fourth and eighth row of Table 2.

Other aspects can also influence the evaluation of an approach. The amount and quality of the required information, the nature and purpose of the problem, and whether the results will be observed by a human or computer must be noticed before committing to an approach (Zeng et al., 2007; Zimmermann, 2000). Non-human observers should be provided with more accurate approximations since they make less flexible judgments, while human observers require better understandable information. Furthermore, a risk scoring process should have a commensurable language when it is to be reviewed by a Peer Review Group (PRG), and on the other hand, a clear rationale to placate potentially dissatisfied stakeholders with threatened interests. Reliance on a popular formula -although able to ensure a high level of commensurability- cannot vindicate the decision to shelve a low RF risk.

Fig. 8 extends the technical discussions provided in Sections 2 and 3 with a guide to constructing efficient formulas tailored to specific project needs, which, in summary, points to parameter selection and preparation, operator selection, and function formation and tuning.

First, considering parameter selection, risk analysis procedures are not usually realized without parameters P and I, as mentioned in Section 3.1, unless for special purposes, when, for example, probability is of little importance, and a specialist provides solutions to a number of defined problems. Also, manageability parameters discussed in Section 3.4 are sometimes useful to highlight those situations that cannot be predicted, or even if predicted there is no time to address, or if predicted timely, they still cannot be prevented. All chosen parameters must therefore be defined clearly, with an explanation as to why they are involved in the process.

Parameters must then be prepared. Here it should be noted that the logarithmic transformation is not necessarily applied to the whole function (i.e. $\log RF = \log(P \times I) = \log P + \log I$), but rather it can be

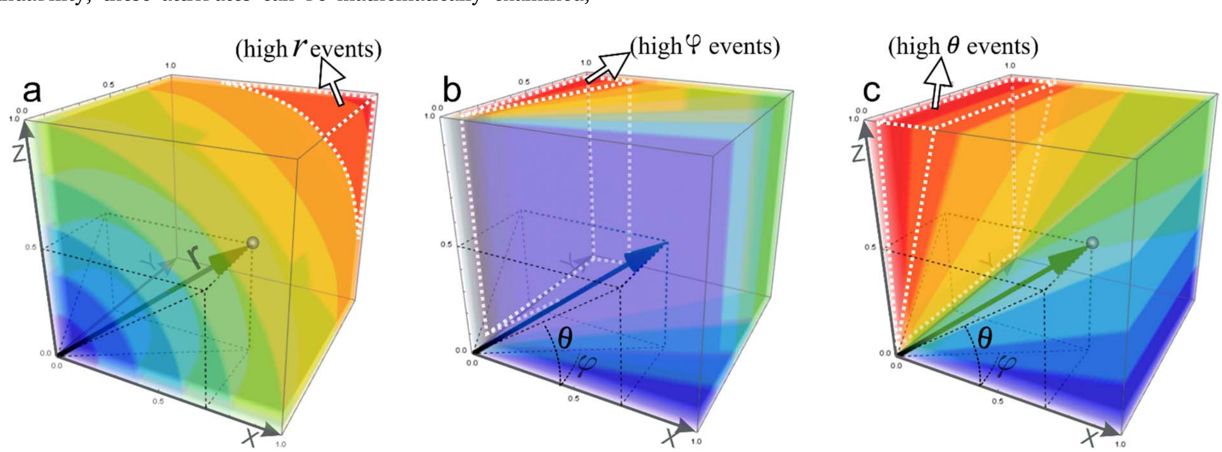

Fig. 7. The 3D visualization of risks in the Cartesian cube.

Table 2
Observed properties of interest in different approaches to risk scoring.

Approach	Ref. section	Original form	Range[a]	Range[b]	Supports dissimilarity	Reasonably Sensitive	Surjective	Confined to [0,1]	Monotonically increasing
Summation/ Average	Section 3.1	$P + I$	[2,20]	[0,2]	No	Yes	Yes	If modified	Yes
Multiplication	Section 3.2	$P \times I$	[1,100]	[0,1]	No	No	Yes	Yes	Yes
Union-Like	Section 3.3	$P + I - P \times I$	*Undefined*	[0,1]	No	No	Yes	Yes	Yes
3Factors/FMEA	Section 3.4	$P \times I \times D$	[1,1000]	[0,1]	No	No	Yes	Yes	Yes
Treatment Efficiency	Section 3.5	$\frac{P \times I}{Cost}$	[0.1,100]	$[0,\infty)$	No	No	Yes	No	No[c]
Logarithmic	Section 3.6	$\log[P \times I]$	[0,2]	$(-\infty,0]$	No	Partly	Yes	No	Yes
Tuned Exponents	Section 3.7	$P^a \times I^b$	$[0,10^{a+b}]$	[0,1]	Yes	No	Yes	Yes	Yes
Multifaceted	Section 4	$\sqrt{P^2 + I^2} \sphericalangle Arctan(\frac{I}{P})$	$[0,10\sqrt{2}]\&[0,\frac{\pi}{2}]$	$[0,\sqrt{2})\&[0,\frac{\pi}{2}]$	Yes	Partly	Yes	If modified	No[c]

[a] Function range when the domain for all parameters is {1, 2, 3, …, 10}.
[b] Function range when the domain for all parameters is [0,1].
[c] The function is still either monotonically increasing or monotically decreasing in each parameter.

used to better present every single parameter that is measured in a logarithmic scale. For example, if the probability statistics are exponentially spaced, while the impact is linearly spaced, it can be advisable to only take the logarithm of the former and keep the latter as it is (e.g. $RF = \log(P) \times I$).

A clear definition of all parameters, achieved at the top right of Fig. 8, facilitates better choices at the bottom left of the chart, where the formula is to be formed. When the summation operator is used (for example in the average formula described in Section 3.1), each operand can increase or decrease the result within a range with a constant length. For example, when RF is defined as $\frac{P+I}{2}$, each parameter has a definite share of 50%, apart from the value of the other one. On the contrary, each of the multiplicands in the product formula can alter the product of all other parameters from 0 to 100%. In other words, they can wipe out the effect of the others, reduce it, or leave it intact. For example, with a P of 0.9 and I of 0.95 out of 1.0, a third parameter D can cut the result of $P \times I \times D$ down to 0, or maintain it at 0.85.

Depending on the nature of the problem, any of the four formulas (average, union-like, product or quotient, and radial distance) can be selected, no matter if the involving parameters are measured logarithmically or linearly. The only condition is that all the operands in the Union-Like formula must be confined to the range [0,1].

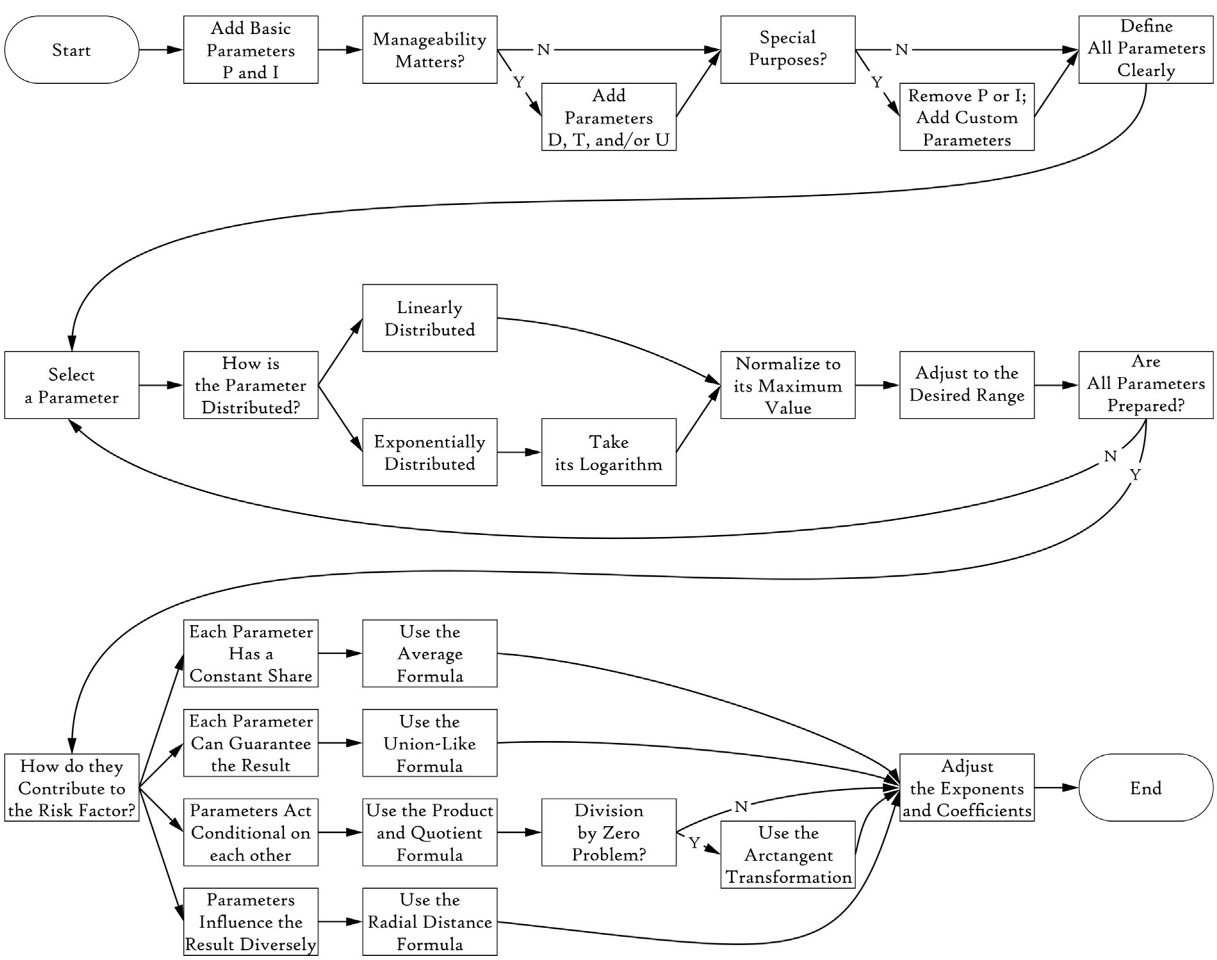

Fig. 8. A procedure to suggest the construction process of a risk factor.

The fourth (radial distance formula) selection box at the bottom of Fig. 8 refers to the magnitude part of the Multifaceted approach presented for the first time in this paper (see Section 4). Defining the magnitude of the RF as a radial distance gives the opportunity of considering each parameter as a new direction in that the situation can move away from a no-risk safe zone, which is placed at the center of the coordinate system. Thus, when two parameters are considered, one moves the point horizontally and the other vertically (Fig. 6), and the radial distance between the risk event and the safe zone is calculated as $\sqrt{P^2+I^2}$, as an example. When there are n parameters involved in the formula, the formula will be written as $\sqrt{\frac{X_1^2+X_2^2+\ldots+X_n^2}{n}}$, after being normalized.

Those formulas that calculate the quotient of two parameters, can then be modified to avoid returning infinity, as it is suggested in the argument part of the Multifaceted approach, φ or θ, using the arctangent transform. This modification is consistent with the assumption of an n-dimensional coordinate system for a risk scoring system that involves n parameters, where the arguments represent the deviation from the axes, and convey information about latent contents of uncertainty (see Section 4).

Functions can also be so tuned by applying coefficients and exponents that the desired attributes are met (see Sections 2.5 and 3.7). Coefficients are usually used to map the results onto a desired range, meanwhile they are useful in tuning a formula that uses the summation operator, in a way quite similar to the way that exponents tune a multiplication formula: They both have to be greater for more important parameters. Exponents greater than one, however, will not behave as expected, as it is shown in Fig. 5. They should all be set lower than one, and the more important a parameter, the closer its exponent to one.

This procedure -explained in a general way by reference to the detailed arguments provided in previous sections-can be repeated as many times as required to provide construction managers with a multifaceted Risk Factor addressing their different needs.

6. Case examples

6.1. Example 1: caring more about impact

The need to distinguish between high Probability and large Impact risks is described in Section 2.1 (Dissimilarity). In this regard, Section 3.7 provides insight into how this can be achieved by better tuning exponents of operands. This example takes into account four cases of the risk of fall from elevation as exemplified in Fig. 9. In the Case A, a worker is standing at a point with low probability of fall, but prone to receive serious injuries if s/he falls from that elevation. In the Case B, a worker stands at an edge with both high Probability and large Impact, in the Case C at a point with high P but small I, and in the Case D at a point with both low P and small I.

The conventional formula ($P \times I$) assigns nearly similar RFs to the Cases A and C. However, although the risk at the Case C should not be ignored, it is necessary to notify managers of the hazard existing at the Case A more than that of the one at the Case C.

As described in Section 3.7, a few previous works have suggested that the exponent of the parameter of more importance should be increased. This research however suggests that this exponent does not exceed 1.0, and in return, recommends that the exponents of less important parameters are reduced. The result of applying these modified formulas on the risk of fall at the Cases A, B, C, and D is summarized in Table 3. For simplicity, the values of P and I for all cases are assumed to be 0.9 when they are 'high' or 'large', and 0.2 when 'low' or 'small'. These assumptions do not retract from the generality of the concept.

As a reminder, the aim is to increase the RF assigned to the Case A, which is only observed in the last column of Table 3.

Moreover, the purpose of differentiating between high Probability and large Impact risks is to better highlight large Impact risks, but not by marginalizing high Probability risks. Fig. 10 demonstrates the RFs calculated in the columns 5 to 7 of Table 3 on a vertical axis. As it can be seen in Fig. 10a, $RF(A)$ and $RF(C)$ are both 0.18. Fig. 10b shows how raising I to a power greater than 1.0 can distinguish between the Cases A and C by marginalizing the 'high P small I' Case C, which is not desired. On the contrary, decreasing the exponent of P −shown in Fig. 10c-better highlights the 'large I low P' Case A, while the RF for Case C is almost maintained at its original value.

This correction will be particularly important when risks are visualized by colors, or when an Acceptable Risk Level (ARL) is defined as a measure to detect whether a risk is to be tolerated or treated. For example, if ARL was defined as 0.3, only the correction suggested by this research would bring the Case A above ARL.

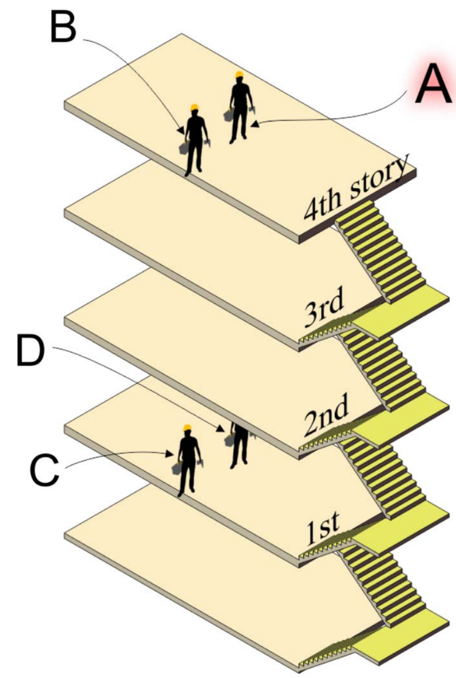

Fig. 9. (Example 1) Four extreme cases of the risk of fall from elevation.

6.2. Example 2: Parameters that diversely influence the result

This example further illustrates the concept behind the selection box located at the bottom left of Fig. 8.

In this example, rather than formulating a Risk Factor, a new parameter -Manageability (M)- is constructed from the combination of the three parameters described in Section 3.4, i.e., Detectability (D), Time (T), and Unpreventability (U).

Theoretically speaking, a risk will be manageable if it is both detectable and preventable, and there is enough time before it happens.

These three parameters cannot be averaged, because they are not of the same nature; cannot be multiplied, because they are not conditional on each other; and cannot be united by the union-like formula, because losing only one of the three leverages of detectability, preventability, and time does not make a risk completely unmanageable. Instead, each of the parameters D, T, and U act in a different direction apart from the value of the others (see Fig. 11).

Table 4 shows the results of calculating M using different formulas selectable at the bottom left of Fig. 8, i.e., the average, product, union-like, and radial distance formulas. In the rows 1 to 4, it can be seen that the product formula returns zero when even only one of the parameters is zero, which is not the case in most situations. In contrast, the same rows show that the union like formula does not distinguish between the cases in which one or more parameters are equal to 1.0; only one

Table 3
(Example 1) Comparing the effect of different approaches to tuning the exponents of involving parameters (see Fig. 10 for a visual demonstration).

(1) Case	(2) The potential accident is	(3) P	(4) I	(5) $P \times I$ (Conventional method)	(6) $P \times I^2$ (Suggested by the literature)	(7) $P^{0.5} \times I$ (Suggested in this research)
A	Not so probable, with large impact	0.2	0.9	0.18	0.16	0.40
B	Highly probable, with large impact	0.9	0.9	0.81	0.73	0.85
C	Highly probable, with small impact	0.9	0.2	0.18	0.04	0.19
D	Not so probable, with small impact	0.2	0.2	0.04	0.01	0.09

parameter of 1.0 guarantees the result of 1.0, i.e., in the union-like formula, losing only one leverage is equal to losing all leverages. The radial distance formula, however, returns different results when inputted with different parameters provided in rows 1 to 4.

In addition to the above mentioned theoretical shortcoming of the average formula, the fifth row of Table 4 shows how a single, low parameter cuts the average of other high parameters, but does not do so that much with their radial distance to the origin, that is, if D and U are 0.9 and 0.8, a T of 0.0 results in a moderate M of 0.57 in the average formula, and a high M of 0.70 in the radial distance formula. In other words, the radial distance formula does not show much sensitivity when a single parameter is considerably less than the others, i.e., the radial distance formula better treats an isolated number than the average formula.

6.3. Example 3: avoiding an irregular distribution of hazards

The following example describes how helpful the new multifaceted approach to risk scoring can be in promoting the concept of Design for Safety.

In a study conducted to assess occupational hazards perceived in a construction site, a fall risk index was defined for each point of a given working area, in the simplest form of which higher indices would be obtained when a point was surrounded by more void areas such as dangerous openings and edges. Probability and Impact could be thus linked to the distance from void areas and their relevant fall height.

Fig. 12 shows an elevated platform with unprotected edges. In the case that a worker loses his/her balance at an arbitrary point W, the probability that this worker falls from point E1 is negatively related to the distance between E1 and W. If the accident occurs, the impact will be positively related to the height of the fall.

Similar reasoning can be made to calculate the risk of falling from other adjacent edges such as points E2 and E3, and a total RF or 'fall risk index' can be assigned to the point W by summing them up.

The procedure to calculate this fall risk index can be simplified as follows:

1. Select the platform which is to be analyzed.
2. Choose the mesh size, 'm'. Meshing is necessary to limit the number of calculations to a bounded value. A mesh size of 0.10 m will be satisfactory for normal purposes.
3. Choose the effective distance, 'd', which shows the horizontal distance a worker might travel after losing his/her balance before hitting the ground. This is again necessary to limit the number of calculations. A distance of 2.0 m is assumed to be enough when external factors such as slope are not involved.
4. Determine the size of the analysis matrices. First, calculate n from $n = d/m$. The analysis matrices will be square matrices of size $2n + 1$.
5. Define the function ' $impact(x,y)$ ' returning a number between 0.0 and 1.0, which represents the impact of fall from a point with coordinates (x,y) on or around the platform.. In general, $impact(x,y)$ is a function of the height of fall, the rigidity and sharpness of the ground, the existence of water, etc. Nevertheless, if the height of fall is constant for all of the edges, this function can be simplified as a

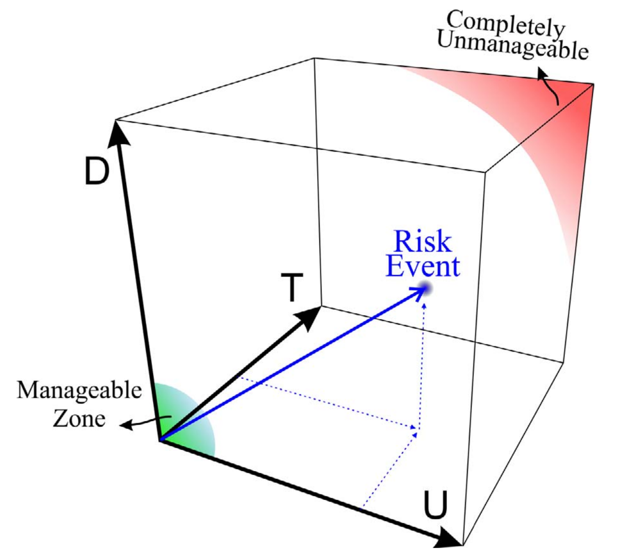

Fig. 11. (Example 2) Each parameter moves a risk event away from the manageable zone in a different direction.

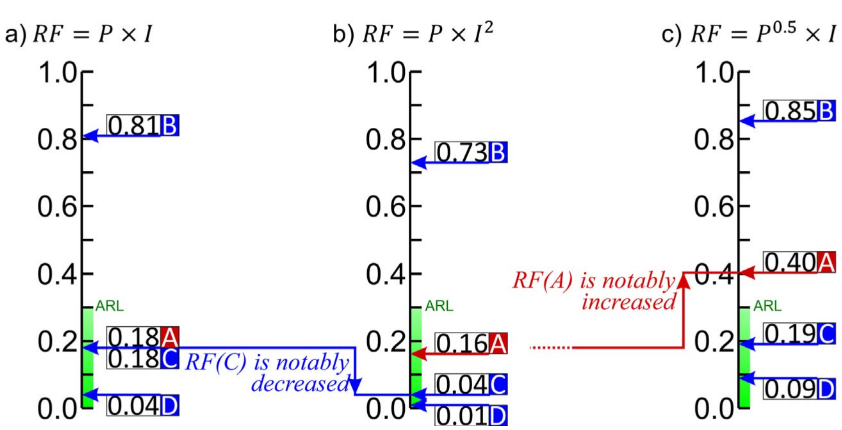

Fig. 10. (Example 1) Demonstration of RF for the Cases A, B, C, and D on a vertical axis calculated by (a) the conventional formula, (b) the recommended formula in the literature, and (c) the formula suggested in this research (see Table 3 for numerical details).

Table 4
(Example 2) Comparison of the results obtained from the four risk scoring formulas introduced in Fig. 8.

Row	D	U	T	Average $M = \frac{D+U+T}{3}$	Product $M = D \times U \times T$	Union-like $M = D + U + T - D \times U - U \times T - D \times T + D \times U \times T$	Radial distance $M = \sqrt{\frac{D^2+U^2+T^2}{3}}$
1	0	0	0	0.00	0.00	0.00	0.00
2	1	0	0	0.33	0.00	1.00	0.58
3	1	1	0	0.67	0.00	1.00	0.82
4	1	1	1	1.00	1.00	1.00	1.00
5	0.9	0.8	0	0.57	0.00	0.98	0.70

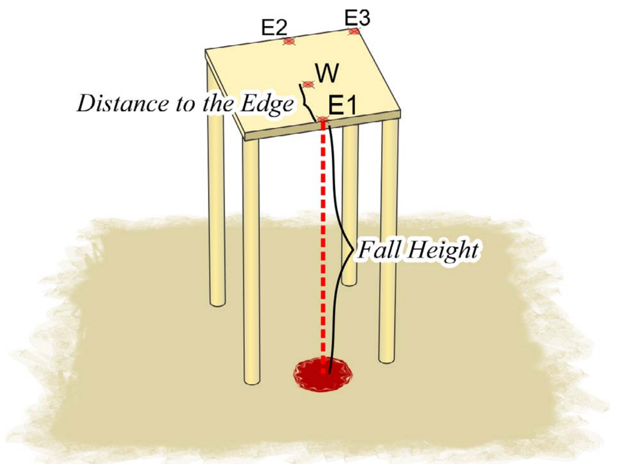

Fig. 12. (Example 3) An unprotected elevated platform. Risk of fall increases both when a worker (point W) approaches an edge (point E1) and when the fall height increases.

piecewise function with two domains:

$$Impact\ (x,y) = \begin{cases} 1.0 & if\ the\ point\ (x,y)\ is\ on\ avoid\ area \\ 0.0 & if\ it\ is\ on\ the\ plat\ form \end{cases}.$$

6. Form the square matrix P of size $2n + 1$ with the elements describing the probability that a worker falls from the point corresponding to $P_{i,j}$ if this worker loses his/her balance at the point corresponding to the element at the center of the matrix (i.e. the element $n + 1, n + 1$ of the $(2n + 1) \times (2n + 1)$ matrix). In the absence of further information, P can be defined using a normal Probability Density Function (PDF) as $P_{i,j} = e^{-m^2 \times ((i-n-1)^2+(j-n-1)^2)}$.[1]
7. Obtain the coordinates of the point for which the fall risk index is to be calculated.
8. Form the square matrix $I(x,y)$ of size $2n + 1$ for the considered point with elements defined as $I_{i,j}(x,y) = impact(x + m \times (i-n-1), y + m \times (j-n-1))$.
9. Form the square matrix $R(x,y)$ of size $2n + 1$ for the considered point with elements defined as $R_{i,j}(x,y) = P_{i,j} \times I_{i,j}(x,y)$.
10. Calculate the Fall Risk Index (FRI) at the considered point as: $FRI(x,y) = \sum_{i=1}^{2n+1} \sum_{j=1}^{2n+1} R_{i,j}(x,y)$

- Resume the analysis by repeating the steps from step 6 to obtain an FRI for every point of the platform.
- Visualize the results of FRI using an orange-red spectrum.

As a practical example, Fig. 13 shows an electrical room with several unprotected wells with similar depths labeled by two digit numbers representing their columns and rows. This electrical room is a simplified derivation of a room designed for a Lighting and Power Substation (LPS) in a subway station, to which different units –including one other LPS at the opposite side of the station- are usually connected through channels situated bellow the rails and accessed via a number of vertical ducts -here referred to as wells. During the construction and procurement phase, workers and specialists are to periodically enter the room to perform various tasks that might include two persons lifting, handling and installing large objects, and extending or retracting cables, all of which involve random movements around specified points.

The above defined fall risk index is visualized in Fig. 14 after performing computer-assisted computations of matrices P, I, and R for every point within the analysis area. As shown in Fig. 14, one to two feet wide red and orange rings are specified as high hazard areas around each well. Looking more closely, one can observe a slight expansion of high hazard areas where two or more wells are clustered together, implying the sense that the region around the clustered wells {31, 32, 41, 42, 51, 52} is of higher risk than around the isolated well 61.

It could be argued, however, that workers are expected to better perceive the risk existing around the clustered wells and seldom or with extreme caution proceed to perform mobility related tasks in such areas. Worker B in Fig. 13, for example, probably takes more care when passing the orange area between the 2nd and 3rd rows of wells.

An isolated well, in contrast, evokes less fear and makes a worker think s/he can perform mobility related tasks in its vicinity without any threat of falling into a well. This relief of stress can sometimes lead to reckless behavior. For example, the workers A and D in Fig. 13 might perceive less risk (from isolated wells 23 and 64) than what the worker C perceives (from the wells 51 and 52 at the left and 61 at the right), and thus, will be probably surprised on confronting the risk of fall into the wells 23 or 64 (see Fig. 15).

As described in Section 4, the surprise content of such an uncertainty is suggested to be determined from the ratio between I and P. In this example, this ratio is simplified by calculating the quotient of the largest impact and the number of points having such impact. Therefore, the multifaceted approach introduced in this paper recommends supplementing the above procedure by the article 11 before resuming to the final steps:

11. Calculate the Risk Surprise Content (RSC) at the considered point as $RSC(x,y) = \frac{Max(I(x,y))}{Count(Max(I(x,y)))}$, in which the maximum value of the matrix $I(x,y)$ formed at step 8 is divided by the number of its appearance in that matrix.

And the final steps will be amended as:

- Resume the analysis by repeating the steps from step 6 to obtain an FRI and an RSC for every point of the platform.
- Visualize the results of FRI using an orange-red spectrum, and the results of RSC using a magenta-purple spectrum.

Having been computed, the two indices FRI and RSC can be visualized on the same graph, which is visualized in Fig. 15.

[1] This PDF is derived from simplifying the Normal PDF $\frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{(x-\mu)^2}{2\sigma^2}}$ by setting μ to 0 and σ to $\frac{\sqrt{2}}{2}$, and multiplying the whole function by $\sqrt{\pi}$ to remove the coefficients and achieve a simple form. Here, $m \times ((i-n-1)^2 + (j-n-1)^2)$ shows the distance between every point addressed by the matrix elements and the central element.

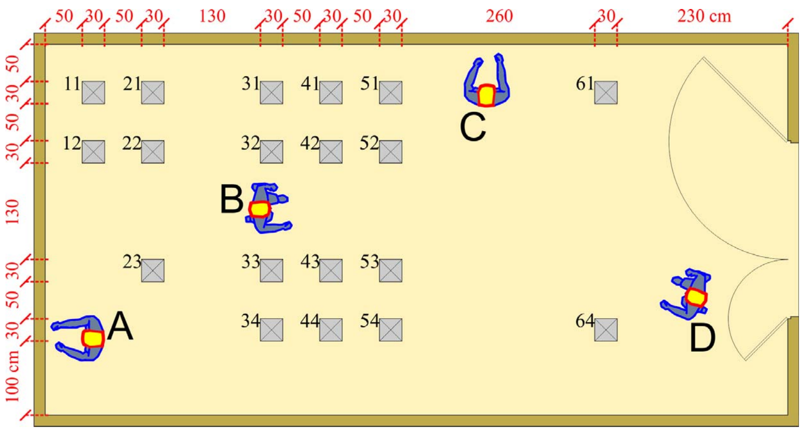

Fig. 13. (Example 3) Plan of the electrical room analyzed against the risk of fall from elevation.

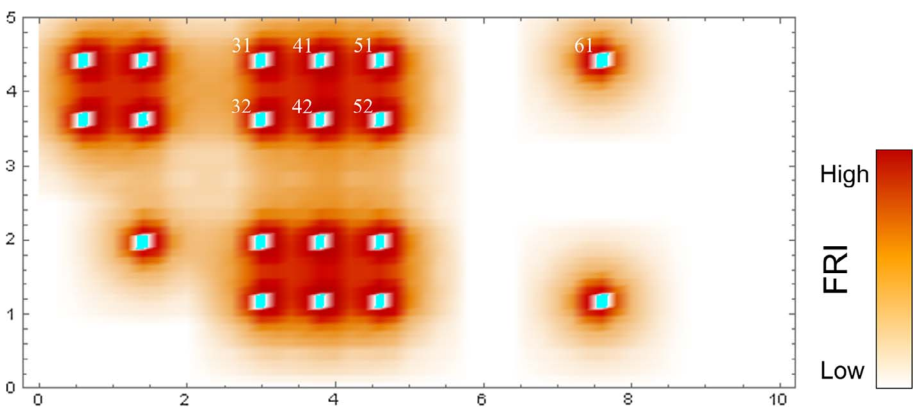

Fig. 14. (Example 3) High hazard areas in the electrical room.

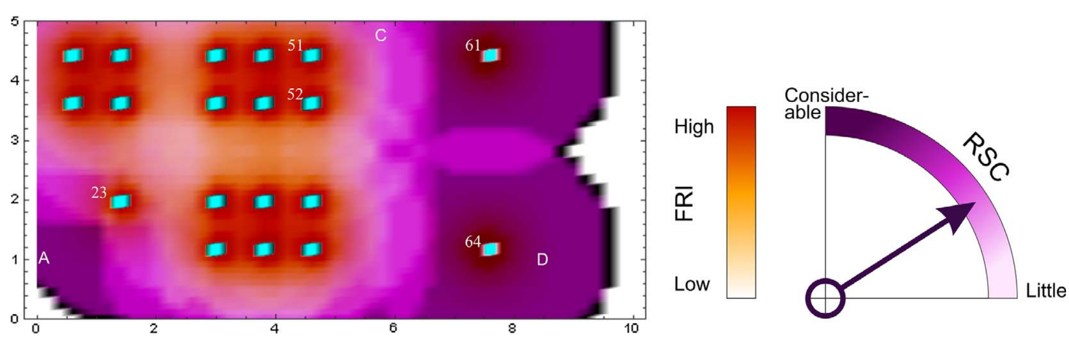

Fig. 15. (Example 3) A multifaceted visualization of risks perceived in the electrical room.

Fig. 15 provides two separate aspects of information about potential fall accidents in the small electrical room. The orange-red spectrum, although sometimes obvious, helps people distinguish high hazard zones, and the magenta-purple spectrum is necessary to acquaint them with the zones where a worker might be in danger while feeling safe.

This practical example is perhaps not inclusive in that it does not address all aspects of fall from elevation and risk perception, but, can emphasize the need for avoiding the placement of high hazards in apparently safe areas at the design phase.

7. Conclusion

The difference between risk scoring formulas is not limited to their attributes; it is rather about how they approach the uncertainty concealed in surrounding events. Constituting elements represent the dimensions perceived as important, and the structure reflects the channel through which uncertain events are believed to affect the objectives of a project.

The three major structures available in the literature, i.e. the average, product, and union-like formula, are completed by the radial distance formula suggested in this paper, while the modifications to these formulas are extended by better tuning of exponents and using new transformations. The radial distance formula, together with the arctangent of the quotient of mutual parameters, is introduced as a new multifaceted risk scoring approach providing the observer with customized contents of uncertainty.

Obtained from different combinations of elements, structures, and modifications, more than ten available forms of scoring formulas have been reviewed in this research with regard to a set of criteria, which are in turn supported by mathematical arguments that unmask fallacious reasoning sometimes used to criticize one approach and promote the other. It is argued that while, in certain situations, some combinations fail -or need some modifications- to produce satisfactory results, there is no unique prescription how to view every possible risk in the construction industry. A solid mathematical foundation, however, makes it possible to embrace a framework offering the proper form of an equation that satisfies given conditions, which is provided in Section 5.

Guidelines on how to select the most relevant elements, useful scales and output ranges, acceptable risk levels, and appropriate exponents and coefficients require separate conceptualizations, which should be developed in future studies.

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